Turbulent Flow-Excited Vibrations of a Shallow Cylindrical Shell

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Turbulent flow-excited random vibrations of a circular, cylindrical, thin, viscoelastic shell, supported at both ends by shear diaphragms, are studied by the normal mode approach. It is shown that the maximum response (area-averaged spectral density of displacements) occurs at the ring frequency and at the level known in literature as the critical frequency for a turbulent flow-excited beam. It is shown that *both* maxima are due to a coincidence effect: equality of the convection velocity of the pressure fluctuations in the turbulent boundary layer, and the phase velocity of the free flexural waves in the shell.

I. Introduction

THIS paper deals with random turbulent excitation of a cylindrical shell supported at both ends by shear diaphragms. In describing the probabilistic behavior of the external pressure, the cross-spectral density of the latter is assumed as a product of longitudinal and circumferential correlation coefficients, in turn taken as functions of the frequency and of the appropriate separation distance only. The representation used for the longitudinal coefficient is chosen so as not to conflict with measurements on the rigid wall in the flow direction. For the circumferential coefficient, two models are used: a fully correlated pressure field, and a modification of the coefficient for the rigid wall in the direction normal to the flow. The formalism of the normal-mode method,[1-3] is used in deriving the probabilistic response characteristics of the shell.

Related works on random excitation of cylindrical shells include Fedoroff,[4] Nemat-Nasser[5] and Kana.[6] The effect of air within the cylinder was taken into account by Bolotin,[7] Hwang and Pi,[8] Bolotin and Elishakoff.[9]

II. Analytical Formulation

Consider a cylindrical shell of uniform thickness, supported at both ends by shear diaphragms and subjected to random pressure fluctuations in a turbulent layer with intensity $q(x,\theta,t)$, x and θ denoting the space coordinates and $t=$time. Assuming that the correlation scale of the pressure is much larger, and the mean-square deviation of the shell displacements much smaller than the thickness of the shell—the problem may be considered in a statistically linear setting. In these circumstances, the normal deflection $w(x,\theta,t)$ must satisfy the Donnell-Vlasov equations[9]

$$D\left(\frac{\partial}{\partial t}\right)\Delta_2\Delta_2 w-\frac{1}{R}\frac{\partial^2\chi}{\partial x^2}+\rho h\frac{\partial^2 w}{\partial t^2}=q(x,\theta,t) \tag{1a}$$

$$\Delta_2\Delta_2\chi+\frac{h}{R}E\left(\frac{\partial}{\partial t}\right)\frac{\partial^2 w}{\partial x^2}=0,$$
$$\Delta_2=\frac{\partial^2}{\partial x^2}+\frac{1}{R^2}\frac{\partial^2}{\partial\theta^2} \tag{1b}$$

where ρ, h and R are respectively the mass density, thickness, and radius of the shell, Δ_2 denotes the Laplace operator on middle surface, $D(\partial/\partial t)$ and $E(\partial/\partial t)$ are viscoelastic operators of the time-differentiation parameter $\partial/\partial t$, corresponding to the stiffness modulus and to Young's modulus, respectively; N_{xx}, $N_{\theta\theta}$ and $N_{x\theta}$ are stress resultants $N_{xx}=R^2\partial^2\chi/\partial\theta^2$, $N_{\theta\theta}=\partial^2\chi/\partial x^2$, $N_{x\theta}=-\partial^2\chi/\partial x\partial\theta$, and χ is the stress resultant function.

Equations (1) are known in literature as those for shallow cylindrical shells or those for stress-strain fields varying rapidly on the middle surface. They are valid for primarily flexural modes of vibration (including the momentless axisymmetric mode); at large wave numbers they have to be corrected for the effects of rotatory inertia and shear deformations.

The supplementary boundary conditions are

$$w=\frac{\partial^2 w}{\partial x^2}=\frac{\partial^2\chi}{\partial x^2}=\frac{\partial^2\chi}{\partial\theta^2}=0 \tag{2}$$

at $x=0$ and $x=\ell$, where ℓ is the shell length.

The problem is formulated as follows: *Given the probabilistic behavior of the ergodic random space-time function $q(x,\theta,t)$, find that of the displacement field of the shell, dispensing with initial conditions.*

For the stochastic boundary-value problem Eqs. (1) and (2), we resort to the normal-mode approach[1-3] which consists in expanding the (probabilistically) given and sought functions in stochastic Fourier integrals and in series in terms of the modes of the undamped shell in vacuo.

The turbulent pressure may be resoved in a stochastic Fourier integral:

$$q(x,\theta,t)=\langle q\rangle+\int_{-\infty}^{\infty}Q(x,\theta,\omega)e^{i\omega t}d\omega \tag{3}$$

where $Q(x,\theta,\omega)$ is the complex random spectrum. The displacement- and force-resultant functions may be represented in the same manner:

$$w(x,\theta,t)=\langle w\rangle+\int_{-\infty}^{\infty}W(x,\theta,\omega)e^{i\omega t}d\omega \tag{4a}$$

$$\chi(x,\theta,t)=\langle\chi\rangle+\int_{-\infty}^{\infty}X(x,\theta,\omega)e^{i\omega t}d\omega \tag{4b}$$

Deviation of the mathematical expectations of the displacements, stresses etc. reduces to solution of the corresponding deterministic problem. Therefore, in the following, we confine ourselves to centered functions:

$$\langle q\rangle=\langle w\rangle=\langle\chi\rangle=0 \tag{5}$$

We formulate the random spectra Q, W and X for each fixed frequency as expansions in terms of the modes of undamped system in vacuo (using complex notation):

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$$Q(x,\theta,\omega)=\sum_{m=1}^{\infty}\sum_{n=-\infty}^{\infty}Q_{mn}(\omega)\psi_m(x)e^{in\theta} \tag{6a}$$

$$W(x,\theta,\omega)=\sum_{m=1}^{\infty}\sum_{n=-\infty}^{\infty}W_{mn}(\omega)\psi_m(x)e^{in\theta} \tag{6b}$$

$$X(x,\theta,\omega)=\sum_{m=1}^{\infty}\sum_{n=-\infty}^{\infty}X_{mn}(\omega)\psi_m(x)e^{in\theta} \tag{6c}$$

where $Q_{mn}(\omega)$, $W_{mn}(\omega)$, and $X_{mn}(\omega)$ are random complex functions of ω, and

$$\psi_m(x)=\sin(m\pi x/\ell),\ m=1,2,3,\ldots \tag{7}$$

Substitution of Eqs. (3-7) in Eqs. (1) yields

$$W_{mn}(\omega)=\frac{Q_{mn}(\omega)}{L_{mn}(\omega)},\quad X_{mn}(\omega)=\frac{Q_{mn}(\omega)}{L_{mn}(\omega)}H_{mn}(\omega) \tag{8}$$

where

$$L_{mn}(\omega)=D(i\omega)\left[\frac{m^2\pi^2}{\ell^2}+\frac{n^2}{R^2}\right]^2$$
$$+E(i\omega)\frac{h}{R}\frac{m^4\pi^4}{\ell^4}\left[\frac{m^2\pi^2}{\ell^2}+\frac{n^2}{R^2}\right]^{-2}-\rho h\omega^2 \tag{9a}$$

$$H_{mn}(\omega)=-\frac{h}{R}E(i\omega)\left[\frac{m^2\pi^2}{\ell^2}+\frac{n^2}{R^2}\right]^{-2} \tag{9b}$$

For the space-time correlation functions of the external pressure

$$K_q(x,\theta,t;x',\theta',t')=<q^*(x,\theta,t)q(x',\theta,t')>(\equiv K_q) \tag{10}$$

(the asterisk denoting the complex conjugate, and the triangular brackets—the expected value), we find

$$K_q=\sum_{m=1}^{\infty}\sum_{n=-\infty}^{\infty}\sum_{p=1}^{\infty}\sum_{q=-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}<Q^*_{mn}(\omega)Q_{pq}(\omega')>$$
$$\times\psi_m(x)\psi_p(x')e^{-in\theta}e^{Iq\theta}e^{i\omega(t'-t)}d\omega d\omega' \tag{11}$$

By the assumption of stationarity of $q(x,\theta,t)$, we have the following condition for statistical orthogonality of $Q_{mn}(\omega)$:

$$<Q_{mn}{}^*(\omega)Q_{pq}(\omega')>=S_{mn,pq}(\omega)\delta(\omega-\omega') \tag{12}$$

[$\delta(\ldots)$ denoting Dirac's delta function, and $S_{mn,pq}$ the cross spectral densities of the generalized forces], whereby Eq. (11) takes the form:

$$K_q=\sum_{m=1}^{\infty}\sum_{n=-\infty}^{\infty}\sum_{p=1}^{\infty}\sum_{q=-\infty}^{\infty}\int_{-\infty}^{\infty}S_{mn,pq}(\omega)\psi_m(x)$$
$$\times\psi_p(x')e^{i(q\theta'-n\theta)}e^{i\omega\tau}d\omega \tag{13}$$

where $\tau=t'-t$ is the time delay.

The Wiener-Khintchine theorem[3] in Eq. (13) permits determination of the cross-spectral density of the external pressure:

$$S_q(x,\theta;x',\theta',\omega)=\frac{1}{2\pi}\int_{-\infty}^{\infty}K_qe^{-i\omega\tau}d\tau(\equiv S_q) \tag{14a}$$

$$S_q=\sum_{m=1}^{\infty}\sum_{n=-\infty}^{\infty}\sum_{p=1}^{\infty}\sum_{q=-\infty}^{\infty}S_{mn,pq}(\omega)\psi_m(x)$$
$$\times\psi_p(x')e^{i(q\theta'-n\theta)} \tag{14b}$$

Equation (14) and the orthogonality property of the eigenfunctions yield the cross-spectral densities of the generalized forces:

$$S_{mn,pq}=\frac{1}{4\pi^2\|\psi_m\|^2\|\psi_p\|^2}\int_0^\ell\int_{-\pi}^{\pi}\int_0^\ell\int_{-\pi}^{\pi}S_q\psi_m(x)e^{in\theta}$$
$$\times\psi_p(x')e^{-iq\theta'}dxd\theta\,dx'[\,d\theta' \tag{15}$$

where

$$\|\psi_m\|^2=\int_0^\ell\psi_m{}^2(x)dx,\qquad \|\psi_p\|^2=\int_0^\ell\psi_p{}^2(x)dx \tag{16}$$

For the correlaton function of the displacements

$$K_w(x,\theta,t;x',\theta',t')=<w^*(x,\theta,t)w(x',\theta',t')>(\equiv K_w) \tag{17}$$

we find

$$K_w=\int_{-\infty}^{\infty}S_w(x,\theta;x',\theta';\omega)e^{i\omega\tau}d\omega(\equiv\int_{-\infty}^{\infty}S_we^{i\omega\tau}d\omega) \tag{18}$$

the cross-spectral density of the displacements being:

$$S_w=\sum_{m=1}^{\infty}\sum_{n=-\infty}^{\infty}\sum_{p=1}^{\infty}\sum_{q=-\infty}^{\infty}\frac{S_{mn,pq}}{L^*_{mn}(\omega)L_{pq}(\omega)}$$
$$\times\psi_m(x)\psi_p(x')e^{i(q\theta'-n\theta)} \tag{19}$$

III. Excitation Cross-Spectral Densities

As established experimentally[11,12] pressure fluctuations in a turbulent boundary layer may be approximated by a random function stationary in time

$$S_q(x,\theta;x',\theta';\omega)=S_q(\xi,\eta,\omega) \tag{20}$$

where $\xi=x'-x$ and $\eta=\theta'-\theta$ are the separation distances in the longitudinal and circumferential directions, respectively. S_q is most commonly represented in product form

$$S_q(\xi,\eta,\omega)=<q^2>\psi(\omega)A(\xi,\omega)B(\eta,\omega) \tag{21}$$

where $<q^2>=S_q$ (0,0,0) is the mean-square value fo the external pressure, and $\psi(\omega)$ the spectral density normalized so that $\int_{-\infty}^{\infty}\psi(\omega)d\omega=1$. The correlation coefficients in the longitudinal and circumferential directions $A(\xi,\omega)$ and $B(\eta,\omega)$ are respectively; generally, they are complex functions of the frequency and of the appropriate separation distance, respectively.

By virtue of the separability assumption, Eq. (21), the cross-spectral densities of the generalized forces Eq. (15), may also be represented as the product:

$$S_{mn,pq}(\omega)=<q^2>\psi(\omega)S_{mp}S_{nq} \tag{22}$$

where

$$S_{mp}=\frac{1}{\|\psi_m\|^2\|\psi_p\|^2}\int_0^\ell\int_0^\ell A(\xi,\omega)\psi_m(x)\psi_p(x')dxdx' \tag{23}$$

$$S_{nq}=\frac{1}{4\pi^2}\int_{-\pi}^{\pi}\int_{-\pi}^{\pi}B(\eta,\omega)e^{i(n\theta-q\theta')}d\theta d\theta' \tag{24}$$

The functions S_{mp} and S_{nq} may be interpreted as the cross-spectral densitites of a corresponding beam-strip of length ℓ and a ring of radius R, respectively.

The function $A(\xi,\omega)$ may be taken in the following form:[13,14]

$$A(\xi,\omega)=e^{-0.265\frac{|\xi|}{\delta_0}}e^{-0.1|\frac{\omega\xi}{u_c}|}e^{-i\frac{\omega\xi}{u_c}} \tag{25}$$

where u_c is the convection velocity and δ_0 the thickness of the boundary layer.

For the function $B(\eta,\omega)$, we use two models: a) a fully correlated pressure field in the circumferential direction with

$$B_I(\eta,\omega) \equiv 1 \tag{26}$$

and b) the correlation coefficient for rigid-wall measurements in the direction normal to the flow, originally reading[13]:

$$B_{II}(\eta,\omega) = e^{-b_\eta(\omega)|\eta|}, \quad b_\eta(\omega) = -0.715\frac{\omega}{u_c} - \frac{2.0}{\delta_0} \tag{27}$$

and modified here, for our purpose, so as to have the following periodicity property:

$$B_{II}(\eta,\omega) = B_{II}(\eta \pm 2\pi k,\omega), \quad k=1,2,3,\ldots \tag{28}$$

The expression for $B_{II}(\eta,\omega)$ is[9]

$$B_{II}(\eta,\omega) = e^{-b_\eta(\omega)|\eta|}e^{2\pi b_\eta(k-1)}$$

$$\text{for } \pi(2k-2) \le |\eta| \le \pi(2k-1) \tag{29a}$$

$$B_{II}(\eta,\omega) = e^{b_\eta(\omega)|\eta|}e^{-2\pi b_\eta(k-1)}$$

$$\text{for } \pi(2k-1) \le |\eta| \le 2\pi k \tag{29b}$$

Now the formula for the cross-spectral density coefficients S_{nq}, Eq. (24) may be specified, taking into account the homogeneity property in the circumferential direction

$$S_{nq} = S_{nn}\delta_{nq}, \quad n,q=0,1,2,\ldots \tag{30}$$

where S_{nq} is the Kronecker delta. The formula for the cross-spectral density of the external forces, Eq. (14), thus becomes

$$S_q = \sum_{m=1}^{\infty}\sum_{n=-\infty}^{\infty}\sum_{p=1}^{\infty} S_{mn,pn}(\omega)\psi_m(x)\psi_p(x')e^{in(\theta'-\theta)} \tag{31}$$

and

$$S_{mn,pn} = S_{mp}S_{nn} \tag{32}$$

The function S_{mp} is given by Eq. (23), and Eq. (24) becomes

$$S_{nn} = \frac{1}{4\pi^2}\int_{-\pi}^{\pi}\int_{-\pi}^{\pi} B(\eta,\omega)e^{in(\theta-\theta')}d\theta d\theta' \tag{33}$$

Equation (33) is valid for both models of the correlation coefficients in the circumferential direction. For the first model, however, further simplification is possible, since we have

$$S_{nn} = \delta_{n0} \tag{34}$$

or, instead of Eq. (32),

$$S_{mn,pn} = S_{mp}\delta_{no} \tag{35}$$

IV. Shell Response Characteristics

We first consider the case

$$S_q(\xi,\eta,\omega) = \langle q^2 \rangle \psi(\omega)A(\xi,\omega)B_I(\eta,\omega)$$

Taking into account the statistical orthogonality conditions, Eqs. (12) and (35), we arrive at the following expression for the cross-spectral density of the displacements:

$$S_w = \sum_{n=1}^{\infty}\sum_{p=1}^{\infty}\frac{S_{mp}}{L_{mo}{}^*(\omega)L_{po}(\omega)}\psi_m(x)\psi_p(x') \tag{36}$$

whereby only axisymmetric modes are excited. The spectral density of the displacements at point (x,θ)

$$S_w(x,\theta,\omega) = S_w(x,\theta;x',\theta',\omega)\Big|_{\substack{\theta'=\theta \\ x'=x}} \tag{37}$$

is given by

$$S_w(x,\theta,\omega) = \sum_{m=1}^{\infty}\sum_{p=1}^{\infty}\frac{S_{mp}}{L_{mo}{}^*(\omega)L_{po}(\omega)}\psi_m(x)\psi_p(x) \tag{38}$$

Concentrating on the area-averaged spectral density, we have

$$s_w(\omega) = \frac{1}{2\pi\ell}\int_{-\pi}^{\pi}\int_0^{\ell} S_w(x,\theta,\omega)dxd\theta \tag{39}$$

$$= \sum_{m=1}^{\infty}\frac{\|\psi_m\|^2}{\ell}\frac{S_{mm}}{|L_{mo}(\omega)|^2} \tag{40}$$

Note that the following simple expression (from Powell[15]) is obtained for the cross-spectral density coefficient S_{mm} from the general formula, Eq. (23), taking into account the homogeneity property of the external pressure in the longitudinal direction

$$S_{mm} = 4\int_0^1 ReA(\zeta\ell,\omega)[(1-\zeta)\cos m\pi\zeta - \frac{1}{m\pi}\sin m\pi\zeta]d\zeta \tag{41}$$

We now attempt to specify the form of the operators $E(\partial/\partial t)$ and $D(\partial/\partial t)$, assuming them to have, for the set of solutions containing the time multiplier $e^{i\omega t}$, the property

$$E(d/dt)e^{i\omega t} = e^{i\omega t}E(i\omega), \qquad E(i\omega) = E_R + iE_I \tag{42a}$$

$$D(\partial/\partial t)e^{i\omega t} = e^{i\omega t}D(i\omega), \qquad D(i\omega) = D_R + iD_I \tag{42b}$$

the dissipation measure [the ratio of the imaginary and real parts of $E(i\omega)$] being generally a function of the frequency $\psi(i\omega) = E_I/E_R$.

If the system is slightly damped, $\psi(i\omega) \ll 1$ and $D_I/D_R \ll 1$, then the area-averaged spectral density has maxima near the roots of the equation

$$Re\, L_{m0}(\omega) = D_R\frac{m^4\pi^4}{\ell^4} + E_R\frac{h}{R} - \rho h\omega^2 = 0 \tag{43}$$

These are the eigenfrequencies of the corresponding undamped shell

$$\omega_{mo} = \left(\frac{D_R}{\rho h}\frac{m^4\pi^4}{\ell^4} + \frac{E_R}{\rho R}\right)^{1/2} \tag{44}$$

As for the function S_{mm} for fixed $m \ne 1$, it has an absolute maximum at the frequencies close to the "convection" of the external pressure $\Omega_m = m\pi/\ell$. When this velocity coincides with the phase velocity of the free flexural waves in the shell, the response of the latter is maximal. This phenomenon is analogous to the familiar conicidence effect in acoustics.[16] For the case under consideration, the coincidence frequency ω_c is determined as the root of the equation

$$Re\, L_{mo}(\omega_c) = 0 \tag{45}$$

Noting that $m\pi u_c/\ell = \omega_c$ and making the appropriate substitution in Eq. (44), we have

$$D_R(\omega_c{}^4/u_c{}^4) + E_R(h/R) - \rho h\omega_c{}^2 = 0 \tag{46}$$

This equation has two roots:

$$\omega_c{}^2 = \tfrac{1}{2}[\omega_{oo}{}^2 \pm (\omega_{oo}{}^4 - 4\omega_o{}^2\omega_{oo}{}^2)^{1/2}] \quad (47)$$

where

$$\omega_o = R^{-1}(E_R/\rho)^{1/2}, \qquad \omega\omega_{oo} = u_c{}^2(\rho h/D_R)^{1/2} \quad (48)$$

denote the ring- and coincidence frequencies for the corresponding beam-strip, respectively.

For $h/R \ll 1$ and/or $u_c{}^2 \gg E/\rho$, these two roots approximate to

$$\omega_{c,1} \simeq \omega_o, \qquad \omega_{c,2} \simeq \omega_{oo} \quad (49)$$

An axisymmetrically flow-excited cylindrical shell thus has two coincidence frequencies, one very close numerically to the ring frequency and the other to the coincidence frequency of the corresponding beam-strip. Note that the beam formally also has two coincidence frequencies, 0 and ω_{oo}. The zero is the counterpart of the ring frequency and obtainable from the latter as the limit for $R \to \infty$; the other may be interpreted as a nontrivial coincidence frequency of the beam strip.

In the case

$$S_q(\xi,\eta,\omega) = \langle q^2 \rangle \psi(\omega) A(\xi,\omega) B_{11}(\eta,\omega) \quad (50)$$

the area-averaged spectral density has the form

$$s_w(\omega) = \sum_{m=1}^{\infty} \frac{\|\psi_m\|^2}{\ell} \frac{S_{mm}S_{oo}}{|L_{mo}(\omega)|^2} + \sum_{m=1}^{\infty} \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{\|\psi_m\|^2}{\ell} \frac{S_{mm}S_{nn}}{|L_{mn}(\omega)|^2} \quad (51)$$

the first sum representing the part of the response due to axisymmetric modes of vibration, and the second part due to nonaxisymmetric ones. These conclusions regarding two nonzero coincidence frequencies are also valid for the first sum. In the general case, the coincidence effect occurs if

$$Re\, L_{mn}(\omega_c) = 0 \quad (52)$$

the counterpart of Eq. (46) here being

$$\omega_c{}^2 - \frac{D_R}{\rho h}\left(\frac{\omega_c{}^2}{u_c{}^2} - \frac{h^2}{R^2}\right)^2 - \frac{E_R}{\rho R^2}\frac{\omega_c{}^4}{u_c{}^4}\left(\frac{\omega_c{}^2}{u_c{}^2} + \frac{h^2}{R^2}\right)^{-2} = 0 \quad (53)$$

with the roots

$$\omega_c{}^2 = \tfrac{1}{2}\Big[-2\frac{h^2}{R^2}u_c{}^2 - \tfrac{1}{2}\omega_{oo}{}^2 \mp \tfrac{1}{2}\omega_{oo}{}^2\left[1 - 4\frac{\omega_o{}^2}{\omega_{oo}{}^2}\right]^{1/2} \mp \Big(2\frac{h^2}{R^2}u_c^2 - \tfrac{1}{2}\omega_{oo}{}^2 \mp \omega_{oo}{}^2 \times \left[1 - 4\frac{\omega_o{}^2}{\omega_{oo}{}^2}\right]^{1/2} - 4\frac{n^4}{R^4}u_c^4\Big)^{1/2}\Big] \quad (54)$$

As in the axisymmetric case, there are again two coincidence frequencies. The higher one is close to ω_{oo} and the lower one not exceeding the ring frequency and for the low circumferential mode numbers n practically equals ω_o. It should also be noted that near this frequency the modal density increases substantially.[17-18]

V. Conclusions

New analytical results on random vibrations of shallow cylindrical shells in a turbulent boundary layer are presented. It was shown that there exist two coincidence frequencies, one closer to the ring frequency and the other to the coincidence frequency in a beam strip of the same length and elastic properties. This finding may account for the minima in the noise reducing behavior of the cylindrical shell, referred to in Refs. 19-22.

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